Cosmological bounds on masses of neutrinos and other thermal relics

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Abstract. With the advent of precision data, cosmology has become an extremely powerful tool for probing particle physics. The prime example of this is the cosmological bound on light neutrino masses. Here I review the current status of cosmological neutrino mass bounds as well as the various uncertainties involved in deriving them. From WMAP, SDSS, and Lyman-α forest data an upper bound on the sum of neutrino masses of 0.65 eV (95% C.L.) can be derived with any assumptions about bias. I also present new limits on other light, thermally produced particles. For example, a hypothetical new Majorana fermion decoupling around the electroweak phase transition must have $m \lesssim 5$ eV.

1. Introduction

Neutrinos are the among the most abundant particles in the universe. This means that they have a profound impact on many different aspects of cosmology, from the question of leptogenesis in the very early universe, over big bang nucleosynthesis, to late time structure formation.

At late times $(T \lesssim T_{\mathrm{EW}})$ neutrinos mainly influence cosmology because of their energy density and, even later, their mass.

The absolute value of neutrino masses are very difficult to measure experimentally. On the other hand, mass differences between neutrino mass eigenstates, (m_1, m_2, m_3), can be measured in neutrino oscillation experiments.

The combination of all currently available data suggests two important mass differences in the neutrino mass hierarchy. The solar mass difference of $\delta m_{12}^2 \simeq 8 \times 10^{-5}$ eV^2 and the atmospheric mass difference $\delta m_{23}^2 \simeq 2.6 \times 10^{-3}$ eV^2 [1–4].

In the simplest case where neutrino masses are hierarchical these results suggest that $m_1 \sim 0$, $m_2 \sim \delta m_{\mathrm{solar}}$, and $m_3 \sim \delta m_{\mathrm{atmospheric}}$. If the hierarchy is inverted [5–10] one instead finds $m_3 \sim 0$, $m_2 \sim \delta m_{\mathrm{atmospheric}}$, and $m_1 \sim \delta m_{\mathrm{atmospheric}}$. However, it is also possible that neutrino masses are degenerate [11–21], $m_1 \sim m_2 \sim m_3 \gg \delta m_{\mathrm{atmospheric}}$, in which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of 2.3 eV (95% conf.) [22]. However, cosmology at present yields a much stronger limit which is also based on the kinematics of neutrino mass.

Very interestingly there is also a claim of direct detection of neutrinoless double beta decay in the Heidelberg-Moscow experiment [23, 24], corresponding to an effective neutrino mass in the $0.1 - 0.9$ eV range. If this result is confirmed then it shows that neutrino masses are almost degenerate and well within reach of cosmological detection in the near future.

Neutrinos are not the only possibility for stable eV-mass particles in the universe. There are numerous other candidates, such as axions and majorons, which might be present. As will be discussed later, the same cosmological mass bounds can be applied to any generic light particle which was once in thermal equilibrium.

Here I focus mainly on the issue of cosmological mass bounds. Much more detailed reviews of neutrino cosmology can for instance be found in [25, 26].

2. Neutrino Decoupling

In the standard model neutrinos interact via weak interactions with e^+ and e^-. In the absence of oscillations neutrino decoupling can be followed via the Boltzmann equation

for the single particle distribution function [27]

$$\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = C_{\text{coll}}, \tag{1}$$

where C_{coll} represents all elastic and inelastic interactions. In the standard model all these interactions are $2 \leftrightarrow 2$ interactions in which case the collision integral for process i can be written

$$C_{\text{coll,i}}(f_1) = \frac{1}{2E_1} \int \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} \frac{d^3\mathbf{p}_3}{2E_3(2\pi)^3} \frac{d^3\mathbf{p}_4}{2E_4(2\pi)^3}$$
$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 + p_4) \Lambda(f_1, f_2, f_3, f_4) S|M|^2_{12 \to 34,i}, \tag{2}$$

where $S|M|^2_{12 \to 34,i}$ is the spin-summed and averaged matrix element including the symmetry factor $S = 1/2$ if there are identical particles in initial or final states. The phase-space factor is $\Lambda(f_1, f_2, f_3, f_4) = f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4)$.

The matrix elements for all relevant processes can for instance be found in Ref. [28]. If Maxwell-Boltzmann statistics is used for all particles, and neutrinos are assumed to be in complete scattering equilibrium so that they can be represented by a single temperature, then the collision integral can be integrated to yield the average annihilation rate for a neutrino

$$\Gamma = \frac{16G_F^2}{\pi^3}(g_L^2 + g_R^2)T^5, \tag{3}$$

where

$$g_L^2 + g_R^2 = \sin^4\theta_W + (\frac{1}{2} + \sin^2\theta_W)^2 \text{ for } \nu_e$$
$$\sin^4\theta_W + (-\frac{1}{2} + \sin^2\theta_W)^2 \text{ for } \nu_{\mu,\tau}. \tag{4}$$

This rate can then be compared with the Hubble expansion rate

$$H = 1.66g_*^{1/2}\frac{T^2}{M_{\text{Pl}}} \tag{5}$$

to find the decoupling temperature from the criterion $H = \Gamma|_{T=T_D}$. From this one finds that $T_D(\nu_e) \simeq 2.4$ MeV, $T_D(\nu_{\mu,\tau}) \simeq 3.7$ MeV, when $g_* = 10.75$, as is the case in the standard model.

This means that neutrinos decouple at a temperature which is significantly higher than the electron mass. When e^+e^- annihilation occurs around $T \sim m_e/3$, the neutrino temperature is unaffected whereas the photon temperature is heated by a factor $(11/4)^{1/3}$. The relation $T_\nu/T_\gamma = (4/11)^{1/3} \simeq 0.71$ holds to a precision of roughly one percent. The main correction comes from a slight heating of neutrinos by e^+e^- annihilation, as well as finite temperature QED effects on the photon propagator [28–41].

3. Neutrinos in structure formation

Given that the neutrino temperature is known, the present contribution to the matter density from massive neutrinos is

$$\Omega_\nu h^2 = N_\nu \frac{m_\nu}{92.5 \text{ eV}}. \tag{6}$$

Thus, even a sub-eV neutrino mass gives a significant neutrino contribution to the energy density and therefore has an effect on structure formation.

Perturbations in the neutrino distribution can be followed by solving the Boltzmann equation [42] through the epoch of structure formation. There are publicly available codes, such as CMBFAST [43], which can do this.

The main difference between neutrinos and cold dark matter is that neutrinos are light and only become non-relativistic around the epoch of recombination, significantly later than matter-radiation equality. While they are relativistic, neutrinos free-stream a distance roughly given by $\lambda = 2\pi/k = c\tau$, where τ is conformal time. The total free-streaming length is therefore roughly given by $\lambda_f \sim c\tau(T = m_\nu)$. At scales smaller than this, the solution to the Boltzmann equation is exponentially damped. For scales larger than the free-streaming length neutrino perturbations are unaffected. It is therefore possible to divide the solution into two distinct regimes: 1) $k > \tau(T = m)$: Neutrino perturbations are exponentially damped 2) $k < \tau(T = m)$: Neutrino perturbations follow the CDM perturbations. Calculating the free streaming wavenumber in a flat CDM cosmology leads to the simple numerical relation (applicable only for $T_{\text{eq}} \gg m \gg T_0$)

$$\lambda_{\text{FS}} \sim \frac{20 \text{ Mpc}}{\Omega_x h^2}\left(\frac{T_x}{T_\nu}\right)^4\left[1 + \log\left(3.9\frac{\Omega_x h^2}{\Omega_m h^2}\left(\frac{T_\nu}{T_x}\right)^2\right)\right]. \tag{7}$$

In Fig. 1 transfer functions for various different neutrino masses in a flat ΛCDM universe ($\Omega_m + \Omega_\nu + \Omega_\Lambda = 1$) are plotted. The parameters used were $\Omega_b = 0.04$, $\Omega_{\text{CDM}} = 0.26 - \Omega_\nu$, $\Omega_\Lambda = 0.7$, $h = 0.7$, and $n = 1$.

When measuring fluctuations it is customary to use the power spectrum, $P(k, \tau)$, defined as

$$P(k, \tau) = |\delta_k(\tau)|^2. \tag{8}$$

The power spectrum can be decomposed into a primordial part, $P_0(k)$, and a transfer function $T(k, \tau)$,

$$P(k, \tau) = P_0(k)T(k, \tau). \tag{9}$$

The transfer function at a particular time is found by solving the Boltzmann equation for $\delta(\tau)$.

At scales much smaller than the free-streaming scale the present matter power spectrum is suppressed roughly by the factor [44]

$$\frac{\Delta P(k)}{P(k)} = \frac{\Delta T(k, \tau = \tau_0)}{T(k, \tau = \tau_0)} \simeq -8\frac{\Omega_\nu}{\Omega_m}, \tag{10}$$

as long as $\Omega_\nu \ll \Omega_m$. The numerical factor 8 is derived from a numerical solution of the Boltzmann equation, but the general structure of the equation is simple to understand. At scales smaller than the free-streaming scale the neutrino perturbations are washed out completely, leaving only perturbations in the non-relativistic matter (CDM and baryons). Therefore the *relative* suppression of power is proportional to the ratio of
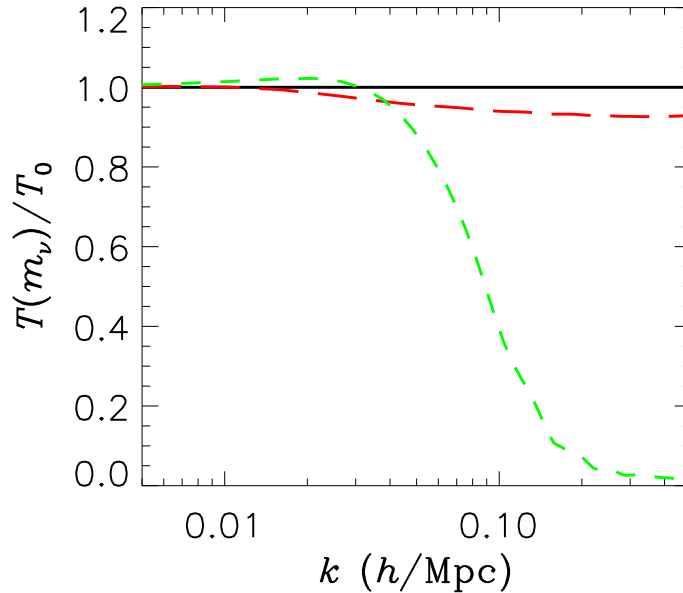
Figure 1. The transfer function $T(k, t = t_0)$ for various different neutrino masses. The solid (black) line is for $m_\nu = 0$, the long-dashed for $m_\nu = 0.3$ eV, and the dashed for $m_\nu = 1$ eV.

neutrino energy density to the overall matter density. Clearly the above relation only applies when $\Omega_\nu \ll \Omega_m$, when Ω_ν becomes dominant the spectrum suppression becomes exponential as in the pure hot dark matter model. This effect is shown for different neutrino masses in Fig. 1.

The effect of massive neutrinos on structure formation only applies to the scales below the free-streaming length. For neutrinos with masses of several eV the free-streaming scale is smaller than the scales which can be probed using present CMB data and therefore the power spectrum suppression can be seen only in large scale structure data. On the other hand, neutrinos of sub-eV mass behave almost like a relativistic neutrino species for CMB considerations. The main effect of a small neutrino mass on the CMB is that it leads to an enhanced early ISW effect. The reason is that the ratio of radiation to matter at recombination becomes larger because a sub-eV neutrino is still relativistic or semi-relativistic at recombination. With the WMAP data alone it is very difficult to constrain the neutrino mass, and to achieve a constraint which is competitive with current experimental bounds it is necessary to include LSS data from 2dF or SDSS. When this is done the bound becomes very strong, somewhere in the range of 1 eV for the sum of neutrino masses, depending on assumptions about priors. This bound can be strengthened even further by including data from the Lyman-α forest and assumptions about bias. In this case the bound on the sum of neutrino masses becomes as low as 0.4-0.6 eV.
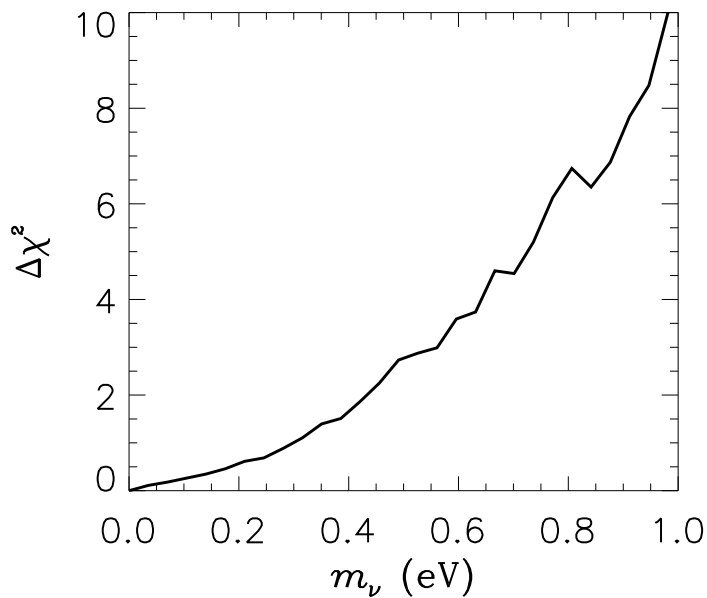
Figure 2. $\Delta\chi^2$ as a function of the the the sum of neutrino masses m_ν.

In Fig. 2 $\Delta\chi^2$ is shown for an analysis which includes the WMAP CMB data [45], the SDSS galaxy survey data [46, 47], the Riess *et al.* SNI-a "gold" sample [49], and the Lyman-α forest data from Croft *et al.* [50]. For the Lyman-α forest data, the error bars on the last three data points have been increased in the same fashion as was done by the WMAP collaboration [45], in order to make them compatible with the analysis of Gnedin and Hamilton [51]. In addition to the neutrino mass, which I take to be distributed in three degenerate species, I take the minimum standard model with 6 parameters: Ω_m, the matter density, Ω_b, the baryon density, H_0, the Hubble parameter, and τ, the optical depth to reionization. The normalization of both CMB, LSS, and Ly-α spectra are taken to be free and unrelated parameters. The priors used are given in Table 1.

Table 1. Priors on cosmological parameters used in the likelihood analysis.

Parameter	Prior	Distribution
$\Omega = \Omega_m + \Omega_X$	1	Fixed
h	0.72 ± 0.08	Gaussian [48]
$\Omega_b h^2$	0.014–0.040	Top hat
n_s	0.6–1.4	Top hat
τ	0–1	Top hat
Q	—	Free
b	—	Free

Table 2. Various recent limits on the neutrino mass from cosmology and the data sets used in deriving them. 1: WMAP data, 2: Other CMB data, 3: 2dF data, 4: Constraint on σ_8 (different in 4^a, 4^b, and 4^c), 5: SDSS data, 6: Constraint on H_0, 7: Constraint from Lyman-α forest.

Ref.	Bound on $\sum m_\nu$	Data used
Spergel et al. (WMAP) [45]	0.69 eV	1,2,3,4^a,6, 7
Hannestad [52]	1.01 eV	1,2,3,6
Allen, Smith and Bridle [53]	$0.56^{+0.3}_{-0.26}$ eV	1,2,3,4^b,6
Tegmark et al. (SDSS) [47]	1.8 eV	1,5
Barger et al. [54]	0.75 eV	1,2,3,5,6
Crotty, Lesgourgues and Pastor [55]	1.0 (0.6) eV	1,2,3,5 (6)
Seljak et al. [56]	0.42 eV	1,2,4^c,5,6,7
The present work	0.65 eV	1,5,6,7

In this particular analysis, the 95% C.L. upper bound on the sum of neutrino masses is 0.55 eV.

In Table 2 the present upper bound on the neutrino mass from various analyses is quoted, as well as the assumptions going into the derivation. As can be gauged from this table, a fairly robust bound on the sum of neutrino masses is at present somewhere around 0.5-1 eV, depending on the specific priors and data sets used.

4. General thermal relics

While light neutrinos are the canonical light, thermally produced particles, there are other species which can have the same properties. At $T \sim M_{\mathrm{Pl}}$ gravitons are presumably in thermal equilibrium. However, since inflation occurs at a lower temperature, there should be no thermal graviton background present.

Other examples include majorons, axions, etc. Any such species which was once in equilibrium is characterized by only two quantities, its mass, m_X, and the temperature at which it decoupled from thermal equilibrium, T_{D}. At this temperature the number of degrees of freedom was g_*. The relevant masses are $\ll$ MeV so that the particles decouple when they are relativistic, i.e. at decoupling they are characterized by a Fermi-Dirac or Bose-Einstein distribution of temperature T_{D}.

In the present-day universe, the new particles will be non-relativistic and contribute a matter fraction

$$\Omega_X h^2 = \frac{m_X g_X}{183 \text{ eV}} \frac{g_{*\nu}}{g_*} \times 1 \text{ (fermions)} \quad \frac{4}{3} \text{ (bosons)} \tag{11}$$

where g_X is the number of the particle's internal degrees of freedom while $g_{*\nu}$ is the effective number of thermal degrees of freedom when ordinary neutrinos freeze out with $g_{*\nu} = 10.75$ in the absence of new particles.

In the late epochs that are important for structure formation, the momentum distribution of the new particles is characterized by a thermal distribution with temperature T_X that is given by

$$\frac{T_X}{T_\nu} = \left(\frac{g_{*\nu}}{g_*}\right)^{1/3}.$$ (12)

In Ref. [57] CMBFAST was modified to incorporate a such thermal relics, both fermionic and bosonic.

Here I present an updated analysis for the fermionic case with $g_X = 2$ (i.e. a single Majorana fermion), using the same data sets as described above. Fig. 3 shows the 68% and 95% likelihood contours for $\Omega_X h^2$ and g_*. Overlayed are isocontours for particle masses.

As an example, a species freezing out at the electroweak transition temperature has $g_* = 106.75$, and from the figure it can be seen that the upper bound on the mass of such a particle is around 5 eV. For a corresponding scalar the mass bound will be slightly higher. For a species decoupling after the QCD phase transition where $g_* \lesssim 20$ the mass bound is roughly $m \lesssim 1$ eV.
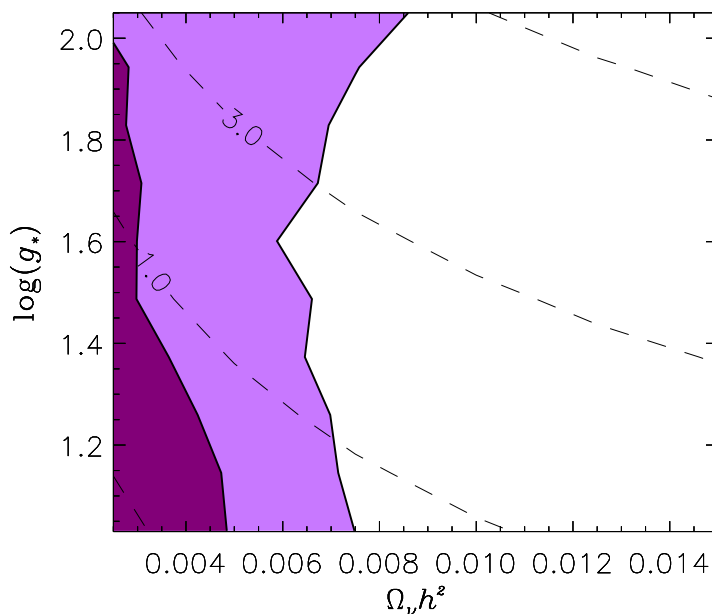

Figure 3. The full lines show 68% and 95% confidence regions in the $(\Omega_\nu h^2, g_*)$ plane for the case where the thermal relic is a fermion with $g = 2$.

5. Conclusion

I have reviewed the present status of cosmological mass bounds on neutrinos and other light, thermally produced particles. Already now these bounds are about an order of

magnitude stronger than current laboratory limits on the neutrino mass, albeit more model dependent. In principle the neutrino mass bound can be evaded either partially or completely by introducing broken scale-invariance in the primordial power spectrum [58] or by coupling neutrino strongly to a massless scalar so that they decay while still semi-relativistic [59].

In the coming years a wealth of new cosmological data will become available from such experiments as the Planck Surveyor CMB satellite. This is likely to allow for a measurement of the (sum of) neutrino masses in the 0.1 eV regime [60–63] . Together with direct detection experiments like KATRIN [64] and future neutrinoless double beta decay experiments, cosmology will provide the answer to whether neutrino masses are hierarchical.

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[64] See the contribution by Christian Weinheimer to the present volume.